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Therefore, using D for $\frac{d}{dx}$, and a' b for a or b ,

$$\lim \frac{u' u'' \dots}{v' v' \dots} = 0' \lim \left(\frac{D^{s'} u' D^{s''} u'' \dots}{s'! s''! \dots} \div \frac{D^{t'} v' D^{t''} v'' \dots}{t'! t''! \dots} \right), \infty,$$

according as $s' + s'' + \dots >$, $=$ or $<$ $t' + t'' + \dots$ (1)

A special case of this problem is to find, dropping the accents, limit $\frac{u^m}{v^n}$, where m and n are any quantities.

We have the identity $\lim \frac{u^m}{v^n} = \lim \left(\frac{u^t}{v^s} \right)^{\frac{n}{s}} \lim u^{\frac{ms - nt}{s}}$, and this is, if no derivative of u or v of not greater order than st is infinite,

$$0' \lim \left[\left(\frac{D^s u}{s!} \right)^m \div \left(\frac{D^t v}{t!} \right)^n \right], \infty, \text{ according as } ms >, = \text{ or } < nt. \quad (2)$$

NOTE — We may find A inductively, using Leibnitz's Theorem. Thus, when $\alpha = a$,

$$D^{s'} u' = \frac{s'!}{s'!} D^{s'} u'$$

$$D^{s'+s''} u' u'' = \frac{(s' + s'') \dots (s' + 1)}{s''!} D^{s'} u' D^{s''} u'' = \frac{(s' + s'')!}{s'! s''!} D^{s'} u' D^{s''} u''$$

$$\begin{aligned} D^{s'+s''+s'''} u' u'' u''' &= \frac{(s' + s'' + s''') \dots (s' + s'' + 1)}{s'''!} D^{s'+s''} u' u'' D^{s'''} u''' \\ &= \frac{(s' + s'' + s''')!}{s'! s''! s'''!} D^{s'} u' D^{s''} u'' D^{s'''} u''' \end{aligned}$$

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EXAMPLES.

$$\begin{aligned} \lim \frac{\sin^3(a-1)[2\cos \frac{1}{2}\pi a + \pi(a-1)][2\cos(a-1) - \cos^2(a-1) + \frac{1}{4}(a-1)^4]}{[(\log a)^2 - (a-1)^2 \div a][2\cos(1-a) + a^2 - 2a - 1][8\varepsilon^{a-1} - \varepsilon^{2a-2} - 2a(a+1) - 3]} \\ = (a=1) \frac{3}{4}\pi^3. \end{aligned}$$

$$\frac{[a-1 - \sin(a-1)]^{\frac{3}{2}}}{[1 - \cos(a-1)]^{\frac{3}{4}}} = (a=1) [\frac{1}{3}\sqrt{2}]^{\frac{3}{2}}; \text{ Ex. 55, chap. 10, Tod. Dif. Cal.}$$

DEMONSTRATION OF THE PROP. AT PAGE 8.

BY PROF. D. J. MC. ADAM, WASHINGTON, PA.

Let $B C D A$ be a semicircle, diameter AB , $ABCD$ being any inscribed trapezium, to prove that the sides AD , DC and CB may represent the reciprocals of the lines a , b , c if AB represent the reciprocal of r .

Draw AC . Then is

$$\begin{aligned} AB^2 &= BC^2 + AC^2 = BC^2 + CD^2 + AD^2 - 2AD \times CD \cos D \\ &= BC^2 + DC^2 + AD^2 + 2AD \times DC \sin CAB \\ &= BC^2 + DC^2 + AD^2 + 2 \frac{AD \times DC \times BC}{AB}. \end{aligned} \quad (1)$$

In a triangle with the given notation,

$$\begin{aligned} \frac{1}{a^2} &= \frac{\sin^2 \frac{1}{2}A}{r^2}; \quad \frac{1}{b^2} = \frac{\sin^2 \frac{1}{2}B}{r^2}; \quad \frac{1}{c^2} = \frac{\sin^2 \frac{1}{2}C}{r^2}; \\ \therefore \quad \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} &= \frac{1}{r^2} \left(\sin^2 \frac{1}{2}A + \sin^2 \frac{1}{2}B + \sin^2 \frac{1}{2}C \right) \\ &= \frac{1}{r^2} \left(1 - 2 \sin \frac{1}{2}A \sin \frac{1}{2}B \sin \frac{1}{2}C \right) \\ &= \frac{1}{r^2} \left(1 - \frac{2r^3}{abc} \right). \end{aligned} \quad (2)$$

Substituting values of $\sin \frac{1}{2}A$ &c.,

$$\frac{1}{r^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{2r}{abc}. \quad (3)$$

Comparing (1) with (3) we find $1 \div r^2$ corresponds to AB^2 , $1 \div a^2$ to BC^2 &c., hence the values of these reciprocals might be made the sides of a trapezium upon AB .

[A demonstration of the above prop. was also sent by Mr. E. B. Seitz.]

SOLUTIONS OF PROBLEMS IN NUMBER TWO.

SOLUTIONS of problems in No. 2 have been received as follows:

From Amateur, 199; R. J. Adcock, 200, 201; Marcus Baker, 196, 197; Prof. W. P. Casey, 196, 197, 200; Prof. P. E. Chase, 197, 200; Geo. M. Day, 196, 197; E. L. De Forest, 201; Capt. J. L. de Fremery, 196, 197; W. E. Heal, 196; H. Heaton, 196, 197, 198, 200; Prof. E. W. Hyde, 201; Chas. H. Kummell, 198, 201; Prof. J. H. Kershner, 196, 197; W. V. Mc. Knight, 196, 197; Prof. D. J. Mc. Adam, 196, 197, 200; Prof. H. T. J. Ludwick, 198; Artemas Martin, 196, 197, 198; K. S. Putnam, 196, 197; P. Richardson, 197; Prof. J. Scheffer, 196, 197, 199, 200; Prof. T. A. Smith, 196, 197; E. B. Seitz, 196, 197, 198, 200; Anna T. Snyder, 197; T. P. Stowell, 197; C. A. Van Velzer, 196, 197, 200.

196. "The sides of a triangle are respectively $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$, x being any number greater than one; prove that the angle opposite the side $x^2 + x + 1$ is equal to 120° ."